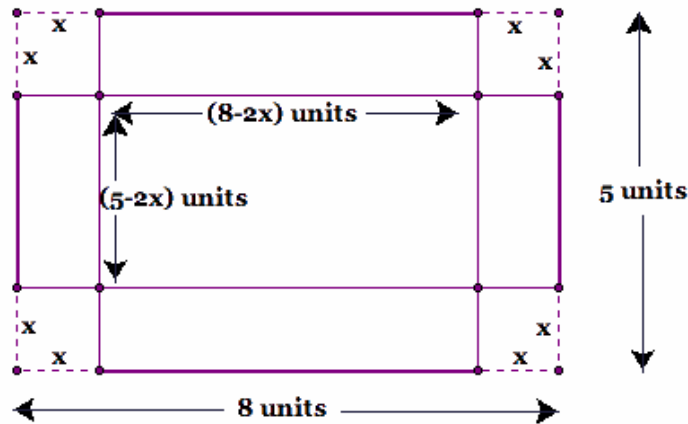# Maximizing the area of Lidless box

Problem: Construct a lidless box from a 5×8 sheet with a square removed from each corner. What dimension maximizes the area?

Solution: Let's consider the lidless box below. We are going to construct an open box by cutting same length from the corner and folding up.

Since the shortest side is 5 in, we can fix the domain as $0 < x < 2.5$. For the purpose of spreadsheet and calculation, I will label as follows: $height = x, width = 5 - 2x$, and $length = 8 - 2x$. The excel sheet below gives us the volume for some differnet combinations:

Height	Width	Length	Volume
x	5-2x	8-2x	x(5-2x)(8-2x)
0.1	4.8	7.8	3.744
0.15	4.7	7.7	5.4285
0.2	4.6	7.6	6.992
0.25	4.5	7.5	8.4375
0.3	4.4	7.4	9.768
0.35	4.3	7.3	10.9865
0.4	4.2	7.2	12.096
0.45	4.1	7.1	13.0995
0.5	4	7	14
0.55	3.9	6.9	14.8005
0.6	3.8	6.8	15.504
0.65	3.7	6.7	16.1135
0.7	3.6	6.6	16.632

0.75	3.5	6.5	17.0625
0.8	3.4	6.4	17.408
0.85	3.3	6.3	17.6715
0.9	3.2	6.2	17.856
0.95	3.1	6.1	17.9645
1	3	6	18
1.05	2.9	5.9	17.9655
1.1	2.8	5.8	17.864
1.15	2.7	5.7	17.6985
1.2	2.6	5.6	17.472
1.25	2.5	5.5	17.1875
1.3	2.4	5.4	16.848
1.35	2.3	5.3	16.4565
1.4	2.2	5.2	16.016
1.45	2.1	5.1	15.5295
1.5	2	5	15
1.55	1.9	4.9	14.4305
1.6	1.8	4.8	13.824
1.65	1.7	4.7	13.1835
1.7	1.6	4.6	12.512
1.75	1.5	4.5	11.8125
1.8	1.4	4.4	11.088
1.85	1.3	4.3	10.3415
1.9	1.2	4.2	9.576
1.95	1.1	4.1	8.7945
2	1	4	8
2.05	0.9	3.9	7.1955
2.1	0.8	3.8	6.384
2.15	0.7	3.7	5.5685
2.2	0.6	3.6	4.752
2.25	0.5	3.5	3.9375
2.3	0.4	3.4	3.128
2.35	0.3	3.3	2.3265
2.4	0.2	3.2	1.536
2.45	0.1	3.1	0.7595
2.5	0	3	0

It looks like 18 cubic inches is the maximum volume of the box implying that the side for the square needs to be cut out at 1 inch.

Now let's confirm our answer by using the plots of Excel. If we treat height (x) as independent variable and volume (V) as dependent variable, we get the graph of scatterplot. The scatter plot shows that the maximum volume is 18 cubic inches.
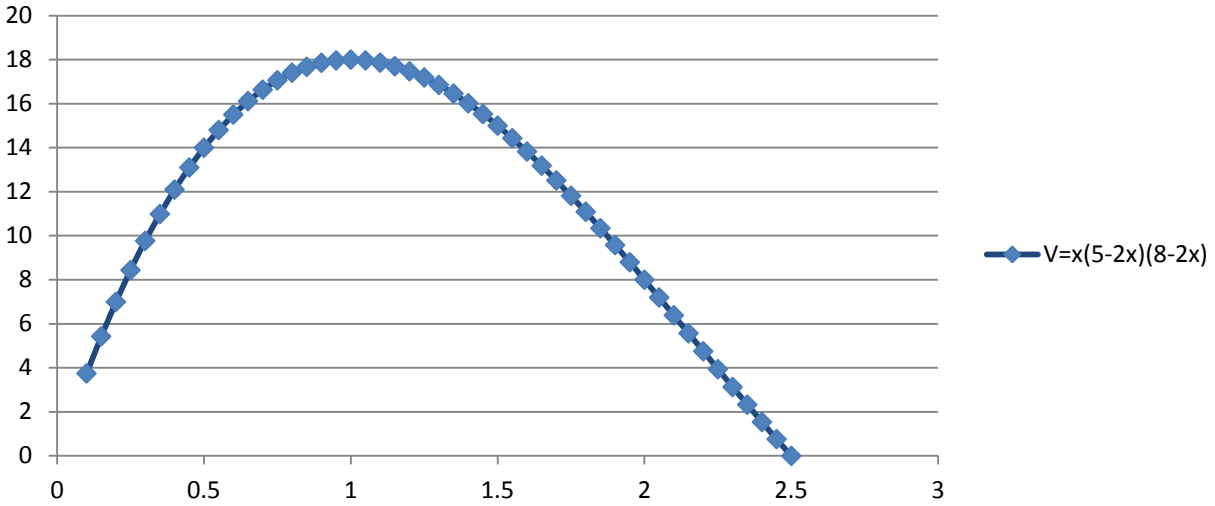
Figure: Relationship between Height and Volume of the box in Scatter Plot

We can also confirm the result using a bar graph on Excel. The bar graph below, again confirms that the maximum volume is 18 cubic inches.
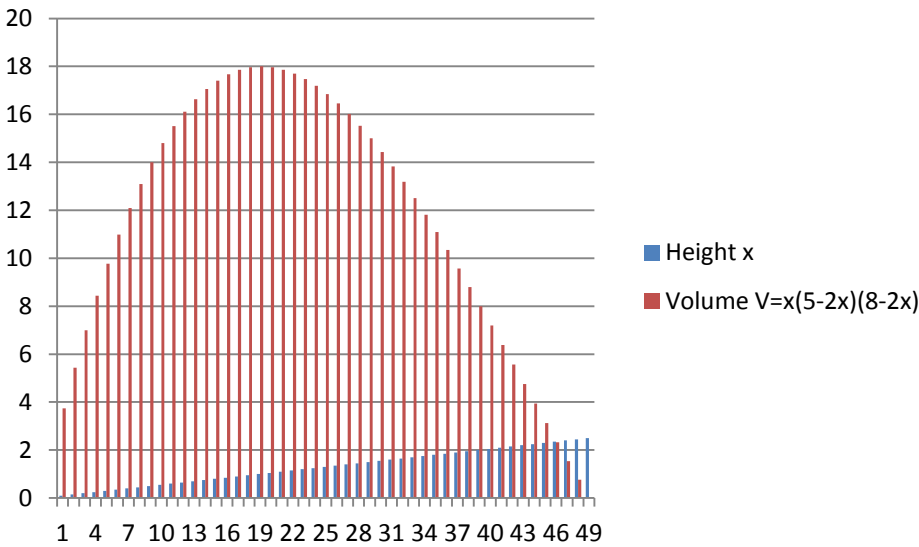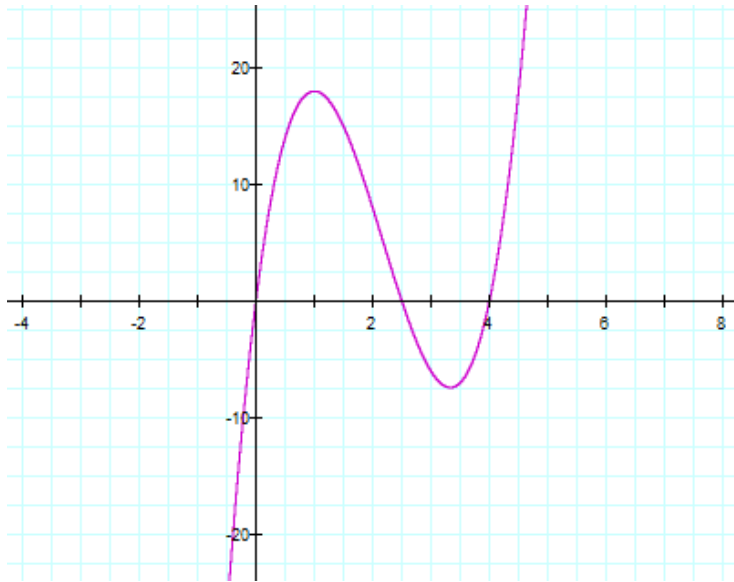

Figure: Relationship between Height and Volume of the box in Box Plot

Now, we can explore the problem through algebra. Let's use the volume formula $v = x(5 - 2x)(8 - 2x)$ to graph the volume equation.

$$y = x(5 - 2x)(8 - 2x)$$

Based on the graph, we can see that the graph has a relative maximum at the point (1, 18). Our investigations have shown us that the maximum value of the box made from a 5x8 sheet with squares cut off at the corners is 18 cubic inches. Furthermore, the 18 cubic inches are formed by the lidless box having a height of 1 inch, a width of 3 inches, and a length of 6 inches.